# Homework Problems

1. A monostatic radar has a system noise temperature of 1000 K and a bandwidth of 2 MHz. The system uses an antenna with a 40 dB gain and a peak transmitted power of 100 kW. If the target has an RCS of 2 m^2, find the maximum range R that will yield a SNR of 12 dB at the receiver. The radar operates at a frequency of 3 GHz.

2. A motorist uses a small microwave receiver to detect the presence of a police radar. The receiver antenna has a gain of 15 dB. The minimum detectable signal level is -60 dBm.
   (a) Find the range at which the police radar be detected assuming it has the same parameters as the example worked in the notes:

   > transmitter power = 100 mW
   > antenna gain = 24.7 dB
   > frequency = 10.55 GHz
   > system noise temperature = 1000 K
   > bandwidth = 1 kHz

   (b) If a superhetrodyne receiver with a bandwidth of 10 kHz and a system noise temperature of 5000 K is used, what is the maximum range at which the police radar will produce a signal such that the SNR is 4? (The resulting range will be extremely large. In practice the range will be limited to a distance slightly greater than the line of sight.)

3. The Laplacian probability density function is given by $p(x) = Ke^{-x/c}$
   (a) Find the constant K so that $p(x)$ is properly normalized.
   (b) Show that the standard deviation is $\sigma = c\sqrt{2}$.
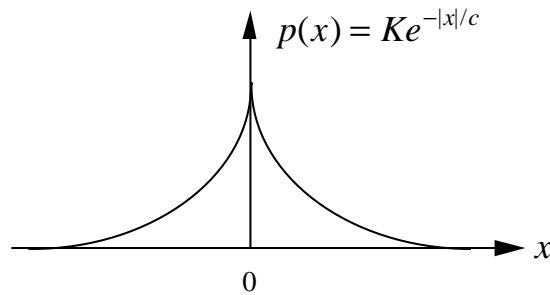


4. The signal plus noise at the output of a certain detector is characterized by a Laplacian distribution shifted by the signal level s, $p(x) = Ke^{-|x-s|/c}$. What is the probability of detection P_d if the threshold V_T is set at $s/2$?
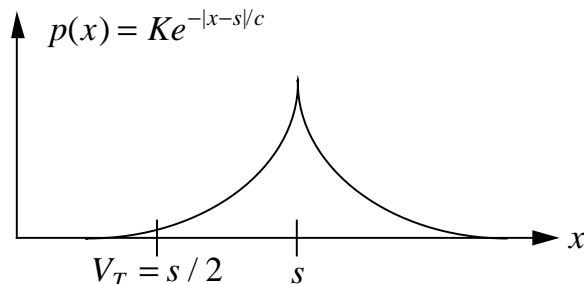
5. Find the Fourier transform of the following:
   (a) a rectangular pulse of width τ shifted in time to t_o,
   (b) a triangular pulse using the fact that the derivative of a ramp is a rectangular pulse.
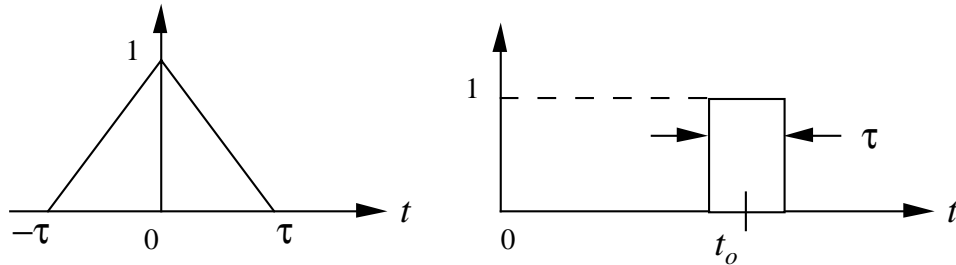
6. The AN/SPS-10 surface search radar has the following operating parameters:

        peak transmitter power = 200 kW
        antenna gain = 33.0 dB
        frequency = 5.6 GHz
        pulse width = 1.4 μs
        PRF = 625 Hz
        antenna scan rate = 16 rpm
        azimuth half-power beamwidth = 1.5 degrees
        antenna noise temperature = 75 K
        receiver noise bandwidth = 1 MHz
        receiver noise figure = 9.88 dB
        system losses ahead of the receiver = 5 dB
        false alarm time = 2 days
   PPI display and operator

   (a) Calculate the number of hits per scan on a point target.
   (b) What is the maximum unambiguous range?
   (c) Find P_{fa} and determine the single pulse SNR in dB required to achieve a detection probability of 0.95.
   (d) Determine the integration improvement in dB (noncoherent integration).
   (e) Calculate the minimum discernable signal power in dBw. (Assume the SNR = 0 dB for this calculation.)
   (f) Calculate the peak effective radiated power in dBw.
   (g) Calculate the effective area of the antenna in square meters.
   (h) Calculate the maximum free space detection range in kilometers for a detection probability of 0.95 (with a false alarm time of 2 days) on a 0 dBsm non-fluctuating point target.

7. Find the backscatter (monostatic radar cross section) of the following objects in square meters and dBsm at 5 GHz:
   (a) looking directly at a circular flat plate with a diameter of 20λ;
   (b) a sphere with a diameter of 5λ;
   (c) a sphere with a diameter of 0.05λ.

8. Scale models are frequently used to verify RCS estimates of full sized targets because they are cheaper and easier to test. The most common scaling technique is called the geometrical model. In this case the model's material composition remains essentially unchanged from the full scale target, but time (frequency) and length are scaled by a factor p. If the full sized target has a length L and the frequency of interest is f, then the scale model length and test frequency should be $L' = L/p$ and $f' = f\,p$. Say that the calculated RCS of a square plate with sides of 10 m at 1 GHz is to be verified by measuring a model plate. The measurement system operates at 10 GHz. What size should the model plate be? (Note: To be precise, the permittivity and permeability remain unchanged in the geometrical model, but the conductivity must be scaled by p. This implies that if the full scale target is a good conductor, the scale model must be an even better conductor. This effect is usually insignificant.)

9. A mixer is used to combine two signals with the same carrier frequency (homodyning)

$$s_1(t) = A_1 \cos(\omega_c t + \Phi_1)$$
$$s_2(t) = A_2 \cos(\omega_c t + \Phi_2)$$

   (c) What frequencies are present at the output of the mixer?
   (d) Write an expression for the baseband signal at the output of the mixer.
   (e) Show that if $A_1 = A_2$ the baseband output can be used to determine the phase difference of between the two input signals. (This is the basis of one type of phase detector or discriminator.)

10. The overall noise figure and gain of a mixer/IF amplifier in a receiver is $F_2 = 4.5$ dB and $G_2 = 12$ dB, respectively. The noise figure of this combination is to be reduced by cascading it with a low-noise amplifier (LNA) having a gain $G_1 = 13$ dB and noise figure F_1. What is the requirement on F_1 to yield a system noise figure less than F_2?

$$F_1, G_1 \qquad\qquad F_2, G_2$$
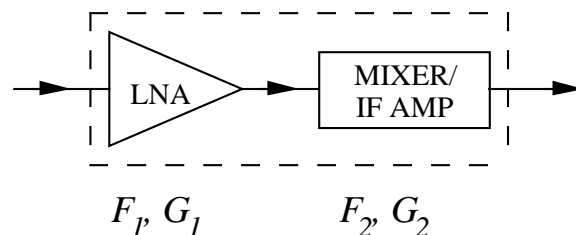
11. A 10 GHz CW doppler radar of the sideband superheterodyne design has the block diagram shown in Figure 3.4 of Skolnik. The radar must be able to measure the speed of vehicles with RCS = 0 dBsm travelling at velocities up to 100 mph at ranges up to 0.5 mile. For reliable operation, a SNR of 15 dB is required at the output of the IF amplifier. The receiver noise figure is 7.25 dB, plumbing loss is 2 dB and the antenna temperature is 300 K. The transmit and receive antennas have gains of 20 dB.

   (a) Calculate the required receiver bandwidth.
   (b) Calculate the required receiver input signal power.
   (c) Calculate the required transmitter output power.

12. Velocity deception is an ECM technique that may be used against doppler tracking radars. The radar signal is received, amplified and retransmitted with the frequency shifted slowly away from the true echo signal doppler frequency. If the radar tracks the false return from the jammer, it will be left with no target when the jammer is turned off.
    (a) A phase modulated RF carrier has the form $v(t) = A \cos[\omega t + \theta(t)]$. Show that when $\theta(t)$ is a sawtooth with peak amplitude of $2\Delta\theta$ and period $1/f_m$, $v(t)$ may be expanded in the Fourier series

$$v(t) = A \operatorname{Re}\left[\sum_{n=-\infty}^{\infty} \frac{\sin(\Delta\theta - n\pi)}{\Delta\theta - n\pi} e^{j2\pi(nf_m + f_c)t} \right]$$

    (b) Calculate the values of $2\Delta\theta$ required to shift all the power from f_c to one of the sidebands.

13. Consider an aircraft flying at 200 m/s with a 10 GHz radar that has an antenna beamwidth of 4 degrees.
    (a) Calculate the ground clutter doppler frequency shift and spectral spread for beam depression angles of 0 and 60 degrees.
    (b) Calculate the PRF required to avoid overlap of clutter regions.
    (c) Calculate the unambiguous range for the PRF you determined in (b).

14. The electric field intensity radiated from an antenna is given by

$$\vec{E}(\theta,\phi) = \frac{e^{-jkR}}{R}\left[\hat{\theta}\cos^n\theta\cos\phi - \hat{\phi}\cos^m\sin\phi \right]$$

    for $|\theta| < \pi/2$. The beamwidth in the $\phi = 0$ plane is determined by the value of n and the beamwidth in the $\phi = \pi/2$ plane is determined by m..
    (a) Find the radiation intensity $U(\theta, \phi)$.
    (b) Calculate the directive gain of this antenna G_D.
    (c) Let $n = 2$ and $m = 1$ and find the gain and the beamwidths in the E- and H-planes ($\phi = 0$ and $\pi/2$ planes, respectively). Compare the gain result to the approximate formula on page 225 of Skolnik: $G_D \approx \dfrac{4\pi}{\theta_E\theta_H}$

15. A CW FM radar operates at 10.5 GHz. The frequency increases at a rate of 2 GHz/s for 990 μs and then returns to its original value in 10 μs.
    (a) What is the frequency shift of the echo from a fixed target at a range of 5000 yards?
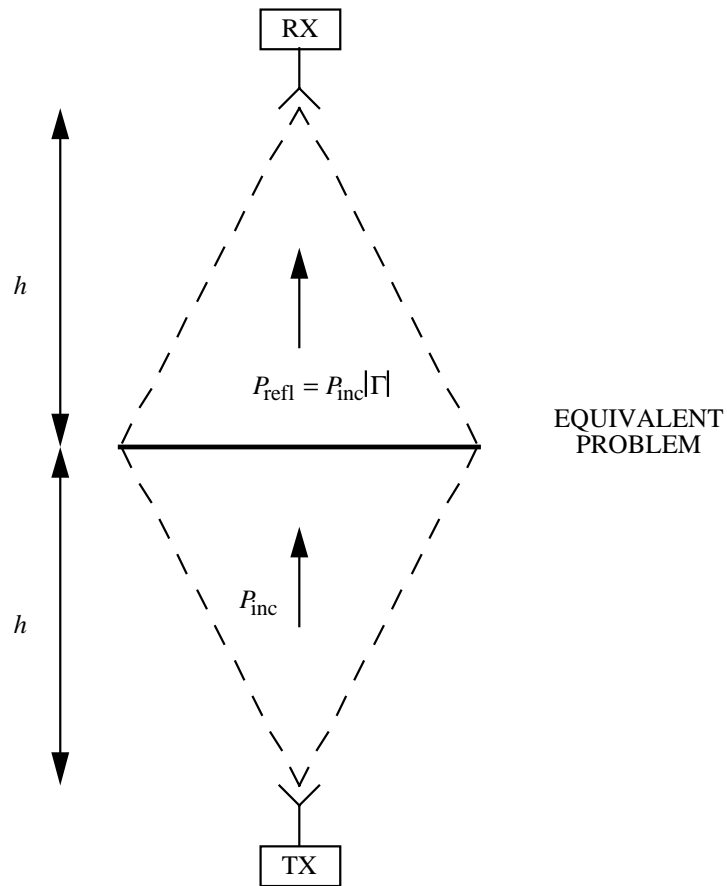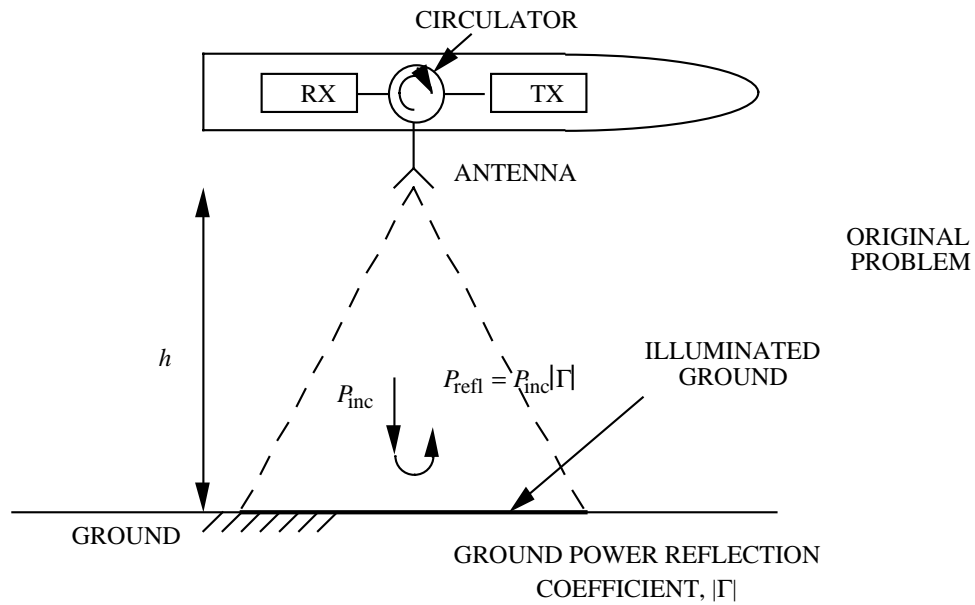   (b) What is the range error due to the doppler shift if the target is closing at a rate of 25 ft/s?

16. A radar altimeter is located a height h above a perfectly smooth flat earth with a surface power reflection coefficient of magnitude $|\Gamma|$. An estimate of the ground clutter signal, which is the target return in this case, can be obtained using the method of images. It states that as far as the receiver is concerned, the reflected radar echo can be considered to originate from an image source a distance h below the surface as shown.
    (a) Derive an equation for the received power based on the method of images.
    (b) If the ground is an isotropic scatterer with a reflection coefficient Γ write an expression for the clutter cross section per unit area, σ^0.

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(c) Show that the result in (a) is identical to beamwidth limited clutter return from isotropic ground when the antenna beamwidth in both principal planes is $\theta_B = \sqrt{2}$ rad.

CIRCULATOR

RX

TX

ANTENNA

ORIGINAL
PROBLEM

h

P_{inc}

$P_{\text{refl}} = P_{\text{inc}}|\Gamma|$

ILLUMINATED
GROUND

GROUND

GROUND POWER REFLECTION
COEFFICIENT, $|\Gamma|$

RX

h

$P_{\text{refl}} = P_{\text{inc}}|\Gamma|$

EQUIVALENT
PROBLEM

h

P_{inc}

TX

17. An antenna measurement system consists of a transmitter with an antenna of known gain G_t. The antenna under test is located a distance R from the transmitting antenna and a received power P_r is measured when the transmitter power is P_t. The test antenna is replaced with a "standard gain horn" which has a peak gain of $G_0 = 18$ dB, and a power P_0 is received. What is the gain of the test antenna in terms of the received powers P_r and P_0 and the reference gain G_0 if the same power is transmitted in both cases?

18. A parabolic reflector antenna with $f/D = 0.4$ is excited by a feed horn whose pattern is approximated by the electric field in problem 13 with $m = n = 2$.
    (a) What is the reflector edge angle, ψ_e?
    (b) Find the illumination edge taper, $\dfrac{\left|\vec{E}(\psi_e)\right|}{\left|\vec{E}(0)\right|}$, in dB.
    (c) Not all of the radiated feed power is intercepted by the reflector. The lost power is called spillover. The spillover efficiency is defined as the ratio of the power collected by the reflector to the total power radiated

$$\text{spillover} = \frac{P_{\text{col}}}{P_{\text{rad}}} = \frac{\Omega_{\text{col}}}{\Omega_A}$$

   where Ω_A denotes the beam solid angle (defined in Skolnik as B) and Ω_{col} is the solid angle subtended by the reflector. Compute the spillover loss for this configuration.

19. A rectangular aperture lies in the x-y plane and has dimensions $D_x = 5\lambda$ and $D_y = 10\lambda$. If the aperture is uniformly illuminated, write an expression for the radiated electric field far from the aperture in terms of the spherical coordinates (θ, ϕ).

20. A half wave dipole lying on the x axis has a radiation pattern in the $\phi = 0$ plane given by

$$g(\theta) = j60I\left[\frac{\cos(\pi\sin\theta/2)}{\cos\theta}\right]\frac{e^{-jkR}}{R}.$$

   (a) An array antenna is comprised of dipoles spaced d_x by d_y. The number of rows and columns is N_x and N_y, and the dipoles are oriented parallel to the x axis. Write an expression for the radiation pattern of the array in the $\phi = 0$ plane using pattern multiplication.
   (b) If the array of dipoles is placed a distance h above a ground plane, use the method of images and the principle of pattern multiplication to find the radiation pattern in the region $|\theta| < \pi/2$ for $\phi = 0$. (Hint: Each dipole has an image and therefore there are two layers of N_x by N_y dipoles. Consider each of these layers to be an element in a two element array.)

21. A randomly thinned array has 20 percent of its elements removed. Estimate the loss in gain relative to a filled array of the same size.

22. A linear array of 16 elements is excited using a parallel feed. There is a phase shifter at each element so that the beam can be scanned.
    (a) How many couplers are required in the feed?

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(b) If the spacing is 0.6λ how far can the beam be scanned before a grating lobe appears in the visible region (i.e., $\theta_g = \pm\,\pi/2$)?

(c) Each coupler has an insertion loss of 0.4 dB, and the phase shifters have a loss of 1.5 dB. What is the total feed loss?

(d) If low noise amplifiers (LNAs) are inserted between the first and second levels of couplers behind the elements, what is the improvement in S/N compared to the array without LNAs?

23. A linear array of equally spaced elements operates at a frequency f. The spacing between elements is 0.5λ at this frequency.

(a) What is the phase shift per element required to scan the beam 45 degrees from the array axis?

(b) If the frequency is increased by a factor of two but the same phase shift per element and spacing is used as in part (a), at what angle will the main lobe be located?

24. An MTI operates at X-band (10 GHz) over the sea. The operating conditions and antenna parameters are such that the clutter doppler shift is spread uniformly over a frequency range of 667 Hz.

(a) What is the corresponding velocity spread in m/s?

(b) What is the RMS velocity spread?

(c) If the radar employs a double canceler without feedback and operates with a PRF of 3300 Hz, what is the approximate improvement factor?

25. A monopulse radar is used to track a metal coated weather balloon with a radius of 1 m. The radar has the following specifications:

> frequency $= 2$ GHz
> transmitted power $= 100$ kW, peak
> Tx/Rx antenna gain $= 30$ dB
> antenna beamwidth, $\theta_B = 5$ degrees
> receiver noise level $= -100$ dBm
> monopulse slope constant $= 1.63\ \theta_B$ (θ_B in radians)

The radar performs coherent integration on 32 pulses. Assume that range independent tracking errors can be neglected. Sketch the RMS thermal-noise error and RMS glint error as a function of range and estimate the range at which the minimum total error occurs.

26. A millimeter wave radar that operates in the presence of clutter has the following requirements:

> signal-to-clutter ratio (S/C) ≥ 15 dB
> track targets with radar cross sections of 0.5 square meters or greater
> measure range to within 1 m at 5 km (RMS)
> azimuth beamwidth, $\theta_{az} = 0.2$ degree
> elevation beamwidth, $\theta_{el} = 1.0$ degree
> operates at low depression angles ($\gamma \approx 0$)
> PRF $= 10$ kHz

When choosing the pulsewidth for this radar there are two reasons to keep the pulse short: (1) to achieve the 1 m (RMS) range error requirement, and (2) to limit the clutter area, A_c, which limits the clutter signal, C.

(a) To what RMS angular accuracy does the 1 m error correspond?
(b) To what RMS transit time error does the 1 m error correspond?
(c) Assume that the peak signal-to-noise ratio is S/C and that the IF bandwidth is the reciprocal of τ. If the time on target (observation time) is 0.04 second, what is the energy-to-noise ratio, E/N_0?
(d) What is the maximum allowable pulse width based on the RMS range accuracy?
(e) The radar operates over the sea where σ^0 = -30 dBsm/sm. If the illumination is pulsewidth limited, what pulsewidth gives S/C = 15 dB at a range of 10 km? (You should find that the requirement for clutter suppression is more critical than the limit due to the range tracking accuracy.)

27. A time waveform is a ramp with negative slope from time 0 to time τ

$$s(t) = \frac{A}{\tau}(\tau - t) \qquad 0 < t < \tau$$

(a) What is the energy in the signal?
(b) What is the matched filter impulse response, $h(t)$?
(c) Assume that $t_1 = \tau$ and find the output of the matched filter for time $t \le \tau$.

28. A radar has a detection range of 200 km without beam shape loss. Calculate the detection range if the beam shape loss (round trip) is 2 dB.

29. A pulse doppler radar uses an antenna with a beamwidth of 1 degree. Find the antenna scan rate for a range rate resolution of 10 ft/sec. Assume a matched filter and wavelength of 5 cm.

30. A radar altimeter uses sinusoidal frequency modulation for ranging.
(a) If the frequency modulation is 200 Hz and the frequency excursion is 60 MHz, calculate the average frequency difference for range increments of 10 and 50 feet.
(b) If the altimeter is required to measure height to within 10 ft, calculate the average incremental frequency change for a modulation frequency of 150 MHz.

31. Calculate the detection range for a search radar that has the following characteristics:

transmitter power = 10 kW, pulse width = 1 μs
PRF = 1000, frequency = 1 GHz
circular antenna diameter = 1 m (efficiency of 70 %)
target radar cross section = 5 m^2
equivalent noise temperature = 1500 K
system losses = 10 dB
detection S/N = 5 dB with 100 pulses integrated
azimuth and elevation search volume = 360 by 20 degrees

32. A medium PRF radar has PRFs of 12, 15, and 18 kHz with f_0 = 13 GHz. Targets appear at 4,

13, and 10 kHz from the centers of the respective PRFs. Find the closing rates.

33. A doppler radar has an antenna HPBW of 1 degree and scan rate of 60 degrees/sec. Assume a coverage of $\pm$ 40 kHz in doppler shift. Using matched filters and a false alarm probability of $P_{fa} = 10^{-10}$ calculate the number of false alarms per minute.

34. A transmitter, antenna, and receiver are connected using a circulator as shown below. The circulator isolation is 50 dB and the insertion loss is 0.5 dB. The receiver minimum detectable signal (MDS) is -10 dBm.
    (a) At what transmitter power level will a received signal be hidden by leakage?
    (b) Assume that the circulator isolation is infinite, i.e., there is no leakage signal from the transmitter to the receiver. At what antenna VSWR will a received signal be hidden by leakage?

TRANSMITTER

ANTENNA

RECEIVER
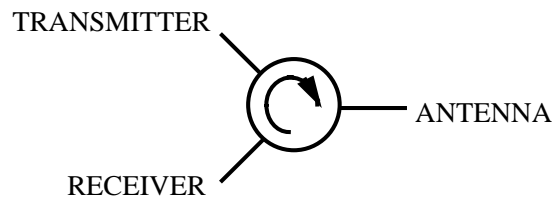
35. A transmission line with a characteristic impedance of $Z_0 = 50$ ohms is connected to an antenna that has an input impedance of $Z_A = 50 + j65$ ohms. Find the antenna VSWR and the percent of power incident on the antenna terminals that is reflected.

36. Write a program to plot a coherent pulse train given by the expression $f(t) = p(t)\cos(\omega_c t)$ where $p(t)$ is the pulse train envelope. Show the result for the following parameters: number of pulses, $N = 3$ pulse width, $\tau = 0.1$ μs carrier frequency, $f_c = 0.1$ GHz pulse period, $T_p = 0.5$ μs.

37. Compute the spectrum of the pulse train of problem 36 by taking the discrete FT. In Matlab this involves using FFT and FFTSHIFT.

38. Modify the program you wrote for problem 36 to plot a noncoherent pulse train by adding a random phase to the start of each pulse

$$f(t) = p(t)\cos(\omega_c t + \Phi_n)$$

where Φ_n is a random variable uniformly distributed from 0 to 2π. In Matlab this requires that you call RAND to obtain N random numbers. Use the program you wrote in problem 36 to the find the spectrum of the noncoherent pulse train.

39. Modify the program you wrote for problem 36 to plot a pulse train with noise by adding a random number each time sample

$$f(t) = p(t)\cos(\omega_c t) + n(t)$$

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where $n(t)$ is a zero mean Gaussian random variable. In Matlab you can use RANDN to generate normally distributed random variables. For a time domain pulse amplitude is 1, try a standard deviation of $\sigma = 0.2$ to generate the noise.

40. An active receive array consists of M identical channels as shown below. The element antenna temperature is T_A, the gain and noise figure of the amplifier are G and F, and the resistance loss factor is L. The signal power arriving at each element is $s = W_i A_{er}$ where A_{er} is the effective receiving area of an element.
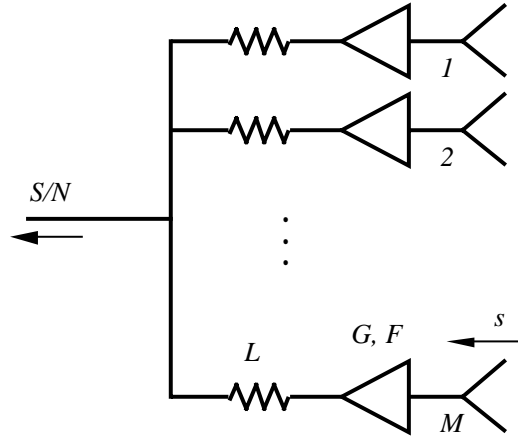
(a) What is the effective temperature of an individual channel (branch) of the feed?
(b) If the noise is uncorrelated white noise, what is the noise power at the output?
(c) Find the signal power at the output and the signal-to-noise ratio, S/N.

41. Monopulse sum and difference beams are formed using the method shown on page II-125. There are 20 elements in each subarray (left and right sides) and the interelement spacing is 0.6λ. The beam formed with the 20 elements on the left side, $AF_1(\theta)$, is scanned to $\theta_s = -1.5°$ and the beam formed with the 20 elements on the right side, $AF_2(\theta)$, is scanned to $\theta_s = 1.5°$.

(a) Write the array factors for the two beams and plot the individual patterns using Matlab.
(b) A sum beam is formed by adding the outputs of the left and right beams:

$$\Sigma(\theta) = AF_1(\theta) + AF_2(\theta)$$

Plot the sum beam using Matlab.
(f) A difference beam is formed by subtracting the outputs of the left and right beams:

$$\Delta(\theta) = AF_1(\theta) - AF_2(\theta)$$

Plot the difference beam using Matlab.
(d) Plot the sum-to-difference ratio Δ / Σ in the vicinity of the null ($|\theta| \le 1.5°$) and estimate the monopulse slope constant, K. (The slope can be found using Matlab POLYVAL.)
(e) Compute the angular accuracy if the SNR is 15 dB.

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42. A high altitude airborne tracking radar is flying towards an oil rig while tracking a low-flying missile. The radar velocity is compensated for by subtracting the doppler shift of the mainbeam echoes. Thus stationary targets in the mainbeam will have zero doppler after processing.
    (a) What is the apparent velocity of the oilrig after processing?
    (b) The RCS of the oilrig, which is in an antenna sidelobe, is 40 dB higher than that of the target. How low must the sidelobe be (relative to the mainbeam gain) so that the target has a 10 dB signal advantage? Assume that both the target and oilrig are at the same range.

43. A 0.5 m by 0.75m antenna in the nose of a fighter aircraft scans a $20°$ by $20°$ frame centered along the aircraft's roll axis. The radar frequency is 10 GHz, the PRF is 10 kHz and 3 bursts (pulse trains) of 32 pulses each are required for detection and confirmation of a target.
    (a) If the antenna is uniformly illuminated find the beamwidths and antenna solid angle Ω_A.
    (b) How many beams are required to scan the frame, and what is the frame time? Assume that the 3 bursts are contiguous.
    (c) If one burst is processed, the probability of detection is $P_d = 0.9$. Given that the cumulative probability of at least one detection in k bursts is $P_{cd} = 1 - (1 - P_d)^k$, how much has the probability of dectection been increased by going to 3 bursts?

44. A SAR looks straight down ($\gamma = 90°$) from an altitude of 15 km. The SAR antenna has a 0.5m uniformly illuminated aperture in the along track direction. The pulse width is 1μs and the wavelength 3.5cm. The radar platform is traveling 90 m/s and the SAR observation time is 6 seconds.
    (a) Compute the azimuth (along track) resolution for a conventional antenna and compare it to those for unfocussed and focussed SAR. What is the range resolution?
    (b) Find the difference in doppler frequency between two points at opposite edges of the footprint. What is the condition on the PRF to avoid a doppler ambiguity?
    (c) What is the condition on the PRF to avoid range ambiguities?
    (d) By combining the results of parts (b) and (c), obtain the condition on the PRF to avoid range and velocity ambiguities.

45. A radar uses a Barker sequence of length four. Denote the uncompressed pulse width by τ_u ($= 4\tau_c$). Returns from two targets overlap by $3\tau_c$ (i.e., there is only τ_c time where the pulses do not overlap). Sketch the output of the "compressor/uncompressor" filter.

46. A radar is operating in the presence of a jammer. The radar has the following parameters:

    transmitter power at the antenna: 1000 W
    frequency: 1GHz
    target RCS: 10 dB
    pulse width: 1 μs
    antenna temperature: 1000 K
    receiver effective temperature: 1000 K

radar processing gain: 20 dB
jammer power: 1 W
jammer range from radar: 20 km
jammer antenna gain: 3 dB

The radar antenna is a rectangular aperture that is pointed at the horizon and rotates 360 degrees in azimuth. The aperture is $w=3$ m wide (azimuth dimension) and $h=1$ m high (elevation dimension) and the efficiency is 80 %. The radiation pattern in azimuth is given by

$$G(\phi_{az}) = \begin{cases} G_o\mathrm{sinc}^2\left(\dfrac{kw}{2}\sin\phi_{az}\right), & 0° \leq |\phi_{az}| \leq 90° \\ G_o\mathrm{sinc}^2\left(\dfrac{kw}{2}\sin\phi_{az}\right)\sin^2\phi_{az}, & 90° < |\phi_{az}| \leq 180° \end{cases}$$

(a) Given the antenna dimensions and efficiency, find G_o, the maximum gain of a uniformly illuminated aperture, and plot the antenna pattern as a function of ϕ_{az}. What is the maximum sidelobe level?
(b) Find the maximum detection range of the radar if there is no jamming. The minimum SNR for detection is $\mathrm{SNR}_{min} = 10$ dB.
(c) Plot the detection range as a function of ϕ when jamming is present. Assume that the minimum signal-to-jammer-plus-noise ratio is $\mathrm{SJNR}_{min} = 10$ dB and that the radar antenna beam maximum is on the target. Therefore, the radar antenna gain in the direction of the jammer is determined from the pattern function above.
(d) Plot the detection range as a function of ϕ when jamming is present if the antenna pattern function is changed to

$$G(\phi_{az}) = \begin{cases} G_o\mathrm{sinc}^4\left(\dfrac{kw}{2}\sin\phi_{az}\right), & 0° \leq |\phi_{az}| \leq 90° \\ G_o\mathrm{sinc}^4\left(\dfrac{kw}{2}\sin\phi_{az}\right)\sin^4\phi_{az}, & 90° < |\phi_{az}| \leq 180° \end{cases}$$

What is the maximum sidelobe level for this pattern?

47. A weather radar has the following parameters:
Antenna azimuth and elevation beamwidths, 2 degrees
Antenna gain, 37dB
Transmitter power, 650 kW
Pulse width, 2 microseconds
Frequency, 2.9 GHz
Noise power, 2×10^{-14} W
(a) What is the range resolution, ΔR?
(b) From the notes (page IV-102), the general formula for a weather radar's volume resolution cell at range R_o is approximately

$$\Delta V \approx \int\limits_0^\infty W^2(R)R_o^2\, dR \int\limits_0^\pi \int\limits_0^{2\pi} \left|\vec{E}_{\text{norm}}\right|^4 \sin\theta\, d\theta\, d\phi$$

Assume a simple antenna model where the pattern is constant between the azimuth and elevation half power beamwidths $(\phi_{az}, \theta_{el}) = (2^\circ, 2^\circ)$

$$\left|\vec{E}_{\text{norm}}\right| = \begin{cases} 1, & -\dfrac{\phi_{az}}{2} \le \phi \le \dfrac{\phi_{az}}{2}, \quad \dfrac{\pi}{2} - \theta_{el} \le \theta \le \dfrac{\pi}{2} \\ 0, & \text{else} \end{cases}$$
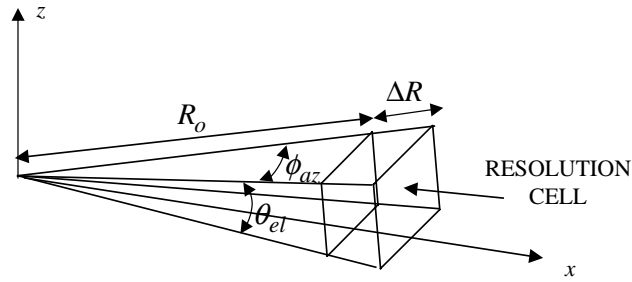
Also assume that the receiver's frequency characteristic is constant in the range cell and zero outside

$$W(R) = \begin{cases} 1, & R_o \le R \le R_o + \Delta R \\ 0, & \text{else} \end{cases}$$

Show that the cell volume is approximately $\Delta V \approx \left(\dfrac{c\tau}{2}\right)(R_o\theta_{el})(R_o\theta_{az})$. (Hint: note that

$\theta \approx 90^\circ$ on the horizon.)

(d) What is the volume of a resolution cell at a range of 20 km in units of m^3?

48. A monostatic ground penetrating radar antenna is tuned for operating on the surface with a gain of 0 dB and a $\sin\theta$ pattern with respect to the downward normal to the surface. An isotropic scatterer (RCS of 0 dBsm) is 2 m below the surface. Plot the received power as a function of distance along the surface if the transmitter power is 100 W and the frequency 50 MHz. Make plots for ground losses of 1, 5 and 10 dB/m.